Why Not More Word Problems?

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"Word problems are a waste of time!" Chairman Toru Kumon said in 1986, in answer to my query about whether or not we should have more word problems in the Kumon Math Program. When I asked him why we had any word problems at all, the Chairman said it was because some parents requested them. The usual justification for his remark in the Kumon community is that word problems do not contribute to the ultimate goal of Kumon math, which is to provide the shortest path to high school calculus.

I never had the chance to learn the Chairman's real reason for his remark, but what follows is my argument for eliminating all word problems from Kumon math worksheets, and for making Kumon math unique among all other math curriculums in the world. The argument consists of two parts:

1. Kumon math teaches mathematics, not the application of mathematics; word problems belong to the application of mathematics.

2. Kumon math introduces ordinal numbers dealing with counting, in worksheet Levels 4A through 2A; most word problems involve cardinal numbers dealing with quantities.

From Level 2A to Level I, 64 worksheets contain word problems (236 in all). These word problems merely provide examples of math's pragmatic utility. The elimination of these worksheets would have no impact on the efficacy and value of Kumon math. The exception is set C 121, where word problems are used to introduce the mathematical concept of remainders.[1] If we were interested in teaching math with real-world applications (word problems), we'd need to develop a separate sequence of worksheets—from simple word problems to complex problems in physics.

Math and Its Applications

The general lack of word problems makes Kumon math unique among math curriculums, differentiating it in particular from the Common Core State Standards, where word problems are a significant part of the K-12 math curriculum. Kumon math teaches *mathematics* (e.g., 2+3=5), and typical school math teaches the *application of mathematics* (e.g., two apples and three apples make five apples).

The Chairman's decision to focus on mathematics and not on application is an important part of the contribution he made in framing the Kumon math method. It resulted in Kumon math's unique features, such as the sequential arrangement of math problems from simple counting to differential equations, a goal-oriented training program with an emphasis on self-learning, and allowed for the possibility of young children learning advanced math.

To understand the Chairman's decision, let's consider the Chairman's background as a mathematician. When Toru Kumon entered Osaka Imperial University in 1933 as a freshman, he studied under Dr. Kenjiro Shoda,[2] a young math professor who had just returned from three years of post-doctoral study at the University of Göttingen in Germany. During his stay in Göttingen, Professor Shoda was surrounded by prominent mathematicians, who were at the center of a modern movement to establish the logical foundation of mathematics. They sought to prove that mathematics is independent of the real world. Their motto was "Mathematics is useful *by accident*." According to their philosophy, "2+3=5" is true by definition, not because it reflects a property of the real world, as prior mathematicians believed.

[1] C 121 will be revised without word problems in 2015. [2] Kenjiro Shoda eventually became the president of Osaka University.

One consequence of this characterization of mathematics is that learning mathematics does not require any knowledge of the real world. Young children can learn mathematics step-by-step before they learn anything about the world. If a baby can count, he can learn addition as repeated counting, subtraction as addition's inverse, multiplication as repeated addition, division as its inverse, fractions as the computation of remainders, and so on. Kumon math, which teaches strictly mathematics, can help even kindergarteners learn algebra. (In my Center, we refer to the children who advance to Level G before they turn six as "super G5" students.)

Another consequence is that learning mathematics is essentially learning the syntactic rules of math expressions. It does not require learning the meaning (or concept) of math expressions (semantics). For example, consider the expression "2+3=5": as long as one can verify this as a "grammatically" correct expression, one need not know what "2" stands for or what "+" stands for.

In learning the applications of mathematics, however, one needs to know the possible interpretation of mathematical symbols in an environment where mathematics is applied. When you use the math fact "2+3=5" to solve a word problem—"How many apples are there when two apples and three apples are added together?"—you have to know that "2" stands for two apples and "+" stands for "added together," and so on. Learning math applications requires both knowledge of mathematics and knowledge of the real world. Because it involves multiple contexts, it is much harder than just learning math.

Kumon Math and School Math

School math curriculums in the United States, based on the Common Core State Standards or its predecessor the NCTM Standards,[3] do not differentiate between math and math applications. They primarily teach math applications (such as word problems, creative thinking, and problem solving), and math itself is a secondary subject. One reason for this emphasis on math application is the educators' belief that students can learn only material *relevant* to their lives, following the education philosophy of John Dewey.[4] Since math *per se* is not relevant to students' lives, teachers assume that it is not an interesting subject for students. Teachers use negative terms like "drill and kill" and "rote memorization" to describe teaching math facts in their classroom. From the point of view of many teachers (and possibly some parents), Kumon abuses children by forcing them to learn material they are not interested in learning.

From our point of view, though, it is common sense to teach mathematics before its application. In high school, you teach calculus before physics. In kindergarten, you should teach "2+3=5" before asking about apples. Kumon students learn mathematics in Kumon and math application in school, while other students must try to learn math application without first learning enough math. Naturally, Kumon students excel in math classrooms. Kindergarten teachers seem to hope that children will eventually learn "2+3=5" after solving enough word problems featuring apples, chairs, and the like. It is a very inefficient and unproductive way of learning mathematics.

Ordinal Numbers and Cardinal Numbers

There is another reason why word problems are not compatible with Kumon Math. The Kumon worksheets of Levels 4A, 3A, and 2A introduce what mathematicians call *ordinal numbers* (first, second, third, . . .), representing ordering and counting. Most word problems, on the other hand, deal with *cardinal numbers* (one, two, three, . . .), representing quantities. In the Arabic numeral system, the same symbols (1, 2, 3, ...) are used to denote ordinal numbers as well as cardinal numbers. Logically speaking, ordinal numbers are much simpler than cardinal numbers,[5] but it has been proved that both numbers share identical mathematical properties. For example, "2+3=5" is valid in cardinal arithmetic as well as in ordinal arithmetic.

The Chairman's knowledge of the logical foundations of mathematics, learned from Professor Shoda, enabled him to introduce the logically simpler ordinal numbers in Kumon math. Other math curriculums in the world introduce cardinal numbers, which are more commonly used in everyday life. In

[3] http://www.nctm.org/standards/

[4] *Democracy and Education: An introduction to the philosophy of education,* 1916.

[5] In a graduate course for computer scientists, it takes only one lecture to define ordinal arithmetic, but it takes one semester to define cardinal arithmetic.

ordinal numbers, "999+1=" (3A 130b) is a simpler problem than "2+2=" (3A 131a), because the former requires one step of counting and the latter requires two steps. In cardinal numbers, however, the former involves a larger (more complex) quantity than the latter. Note that in the worksheets, from 3A 74 to 3A 130, "+1" does not mean the addition operation, but a counting operation to find the next number. The Kumon worksheets of Level 3A and Level 2A are so unique that no other math curriculum can compete with them.

Starting in 3A 131a, the addition operation (+) of ordinal numbers is introduced as repeated counting, which is mathematically defined by the following two rules:[6]

Rule (1) $a+0 = a$
Rule (2) $a+(b+1) = (a+b)+1$

where a and b are arbitrary ordinal numbers, and "+1" is the operation of finding the next number, not the addition operation.

We can show that "2+3=5" is *true by definition*, by deriving from these two rules:

$2+3$	$=2+(2+1)$	by definition of $3=2+1$
	$=(2+2)+1$	by Rule (2)
	$=4+1$	assuming that $2+2=4$
	$=5$	by definition of $5=4+1$

Note that "2+2=4" can be derived similarly from rules (1) and (2).

From this demonstration, it is clear that in order to verify "2+3=5," one merely follows the syntactic definition of "+" and it remains unnecessary to know the concept of "+," whatever it is. When schoolteachers complain about Kumon students knowing math facts but not understanding the concepts, they are confusing math (syntax) and math applications (semantics).

Since "2+3=3+2" is obvious in cardinal arithmetic, but not so in ordinal arithmetic, some parents are puzzled by the fact that a Kumon student could know "3+2=5," but not "2+3=5."

The fact that Kumon math teaches ordinal numbers is also demonstrated in worksheet 2A 71a (among others). It suggests that "+6" can be derived from "+5":

2A 71a(1) $2+5$	$=7$	
2A 71a(2) $2+6$	$=2+(5+1)$	by definition of $6=5+1$
	$=(2+5)+1$	by Rule (2)
	$=7+1$	from 2A 71a (1), $2+5=7$
	$=8$	by definition of $8=7+1$

In summary, Kumon math is designed to teach mathematics and not the application of mathematics (word problems). Since most word problems involve cardinal arithmetic, and Kumon math teaches ordinal arithmetic, word problems are strangers in the Kumon math worksheets.

One of the reasons why this "New Math" of the 1960s failed was that it included an attempt to teach cardinal arithmetic using set theory, which is logically much more complex than teaching ordinal arithmetic. Today, the Common Core has its own difficulty because it attempts to teach application of mathematics without teaching mathematics.

The Chairman started Kumon math as a logically sound curriculum for teaching mathematics proper. By listening to and learning from Instructors and students, he continued revising and improving the math worksheets to be more effective and enjoyable for children as they self-learn. There is no comparable curriculum in the world, so far. ■

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[6] Halmos, Paul. *Naive Set Theory*. Princeton, NJ: D. Van Nostrand Company, 1960.